Grade IX

Lesson : 6 LINES AND ANGLES   BASIC TERMS AND DEFINITIONS

Line → Having no end points

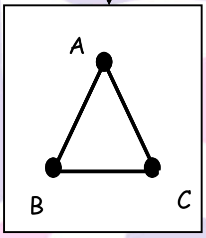
Line → Having one end point (A, B)

Line segment → Having two end point (A, B)

Point → A point is the dot which has no dimensions

Collinear point → Three or more points lie on the same line → A B C D

Non-Collinear point → Three or more points do not lie on the same line →



Types of Angles

An angle is formed by two rays that have common end point [vertex]

θ

Obtuse angle A θ B C $90^0 < \theta < 180^0$

Suplementary angles two angles whose sum is 180^0

Complementary angles Two angles whose sum

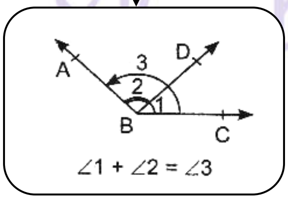
Straight Angle $\theta < 180^0$ θ A O B

Right angle $\theta = 90^0$ A θ B C

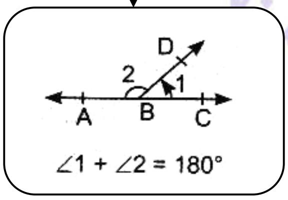
Reflex angle $\theta = 360^0 - x^0$ $180^0 < \theta < 360^0$ θ x

Acute angle $0^0 < \theta < 90^0$ A θ B C

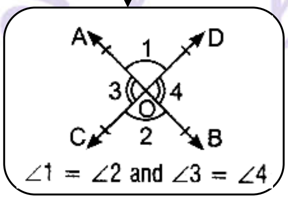
Adjacent angles



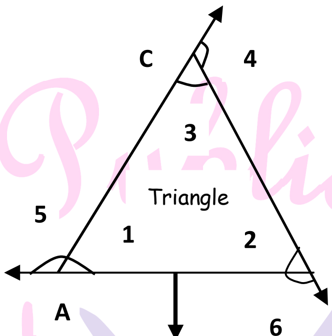
Linear pair angles



Vertically opposite angles



ANGLE SUM PROPERTY OF TRIANGLE AND EXTERIOR ANGLE PROPERTY



Angle sum property

For interior angles
(Angles inside triangle)
$\angle 1 + \angle 2 + \angle 3 = 180^0$

For exterior angles
Angle between one side of a triangle and the extension of an adjacent side.

Exterior angle = sum of two interior opposite angles

$\angle 4 = \angle 1 + \angle 2$; $\angle 5 = \angle 2 + \angle 3$; $\angle 6 = \angle 1 + \angle 3$ and $\angle 4 + \angle 5 + \angle 6 = 360^0$

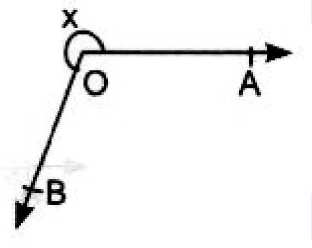
- The sum of the angles of a triangle is 180^0
- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.
- Exterior angle of a triangle is greater than either of its interior opposite angles

Objective Type Questions

I. Multiple choice questions 6.1

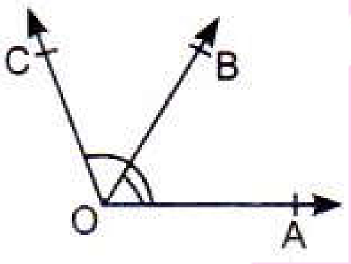
1. In the given figure, $\angle x$ is

a) obtuse angle b) acute angle c) reflex angle d) straight angle



Sol. c

2. In the given figure, a pair of adjacent angles is

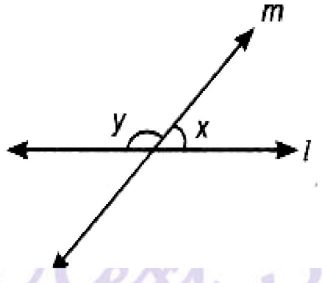


a) $\angle COA$ and $\angle BOA$ b) $\angle COA$ and $\angle BOC$ c) $\angle AOB$ and $\angle BOC$ d) *none of these*

Sol. c

3. In figure if x : y = 1 : 4, then values of x and y are respectively

a) 36^0 and 144^0 b) 18^0 and 72^0 c) 144^0 and 36^0 d) 72^0 and 18^0



Sol. Given, x : y = 1 : 4

$\Rightarrow \quad \frac{x}{y} = \frac{1}{4} = \frac{K}{4K} \Rightarrow x = K \text{ and } y = 4K$

From the figure,

$x + y = 180^0$ (Linear pair axiom)

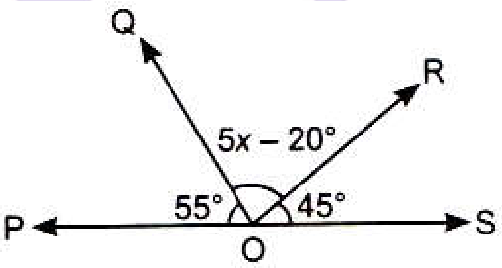
$\Rightarrow \quad k + 4k = 180^0 \Rightarrow 5k = 180^0 \Rightarrow k = 36^0$

Hence, x = k = 36^0

And y = 4k = 4 x $36^0 = 144^0$

$\therefore$ *Correct option is (a)*

4. **In the given figure, POS is a line, then $\angle QOR$ is**



a) 60^0 b) 40^0

c) 80^0 d) 20^0

Sol. c

5. **If the difference between two complementary angles is 10^0 then the angles are,**

a) $50^0, 60^0$ b) $50^0, 40^0$ c) $80^0, 10^0$ d) $35^0, 45^0$

Sol. Let an angle be x. Then other angle = x-10^0.

Since the two angles are complementary, So,

$x + x - 10^0 = 90^0$

$\Rightarrow \quad 2x = 90^0 + 10^0 = 100^0$

$$\Rightarrow \quad x = \frac{100^0}{2} = 50^0$$

So, one angle = 50^0, Then, other angle = x - 10^0 = $50^0 - 10^0 = 40^0$

$\therefore$ ***Correct option is (b)***

6. **Diagonals of a rhombus ABCD intersect each other at O, then, what are the measurements of vertically opposite angles $\angle AOB$ and $\angle COD$?**

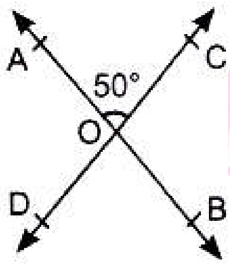
a) $\angle ABO = \angle CDO$ b) $\angle ADO = \angle BCO$ c) $60^0, 60^0$ D) $90^0, 90^0$

Sol. d

7. **In the given figure, if $\angle AOC = 50^0$, then ($\angle AOD = \angle COB$) is equal to**

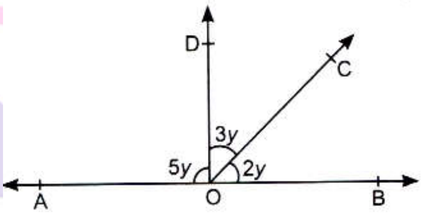
a) 60^0 b) 140^0 c) 260^0 d) 130^0

Sol. c



8. **In the given figure, if AOB is a line then find the measure of $\angle BOC, \angle COD, and\ \angle DOA$**

[CBSE 2011]



Sol. We have, $\angle BOC + \angle COD + \angle DOA = 180^0$

$\Rightarrow \quad 2y + 3y + 5y = 180^0$

$\Rightarrow \quad 10y = 180^0 \Rightarrow y = 18°$

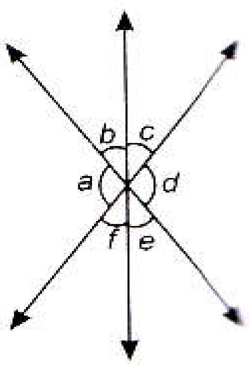
$\therefore \angle BOC = 2y = 2 \times 18^0 = 36^0$

$\angle COD = 3y = 3 \times 18^0 = 54^0$

$\angle DOA = 5y = 5 \times 18^0 = 90^0$

9. **Check whether the following statements are true or not?**

(i) a + b = d + c (ii) a + c + e = 180^0 (iii) b + f = c + e



Sol. From figure, we have

a + b = d + e (Vertically opposite angles)

But $e \neq c$

$\therefore \quad a + b \neq d + c$

$\Rightarrow \quad Hence, statement\ (i)\ is\ incorrect.$

From the figure, we have

a + f + e = 180^0 [Linear pair axiom] .....(i)

But f = c (Vertically opposite angles)

$\Rightarrow$ a + c + e = 180^0 [From (i)]

$Hence, statement\ (ii) is\ true..$

Again b + c = f + e [Vertically opposite angles]

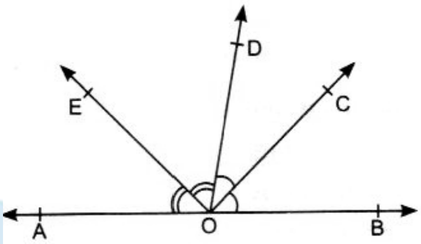
But, c = f [Vertically opposite angles]

B + f = c + e [On interchanging c and f]

Hence, statement (iii) is also true.

Therefore, statement (ii) and (iii) are correct.

10. Ray OD stands on line AOB, if ray OC and OE bisects $\angle BOD$ and $\angle AOD$, respectively, Find the $\angle COE$.



Sol. We have $\angle BOC = \angle COD = \frac{1}{2} \angle BOD$

($\because$ $OC\ is\ angle\ bisector\ of\ \angle BOD$)

And $\angle AOE = \angle EOD = \frac{1}{2} \angle AOD$

($\because$ $OE\ is\ angle\ bisector\ of\ \angle AOD$)

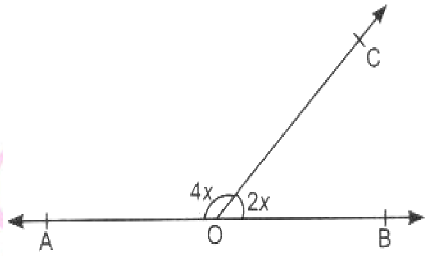
Now, $\angle AOD + \angle BOD = 180^0$ [Linear pair axiom]

$\Rightarrow \frac{1}{2} \angle AOD + \frac{1}{2} \angle BOD = \frac{1}{2} \times 180^0 = 90^0$

$\Rightarrow \quad \angle EOD + \angle COD = 90^0$

$\Rightarrow \quad \angle COE = 90^0$

11. In the given figure, find the value x,



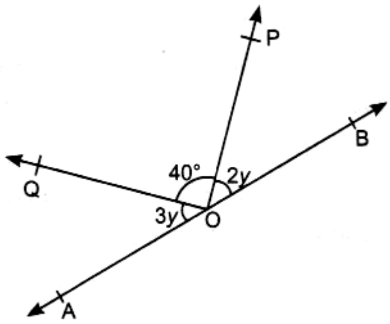
Sol : Ray OC stands on line AOB

$\therefore\ \angle AOC + \angle BOC = 180^0$ [Linear pair axiom]

$\Rightarrow \qquad 4x + 2x = 180^0$

$\Rightarrow \qquad 6x = 180^0 \Rightarrow x = \frac{180^0}{6} = 30^0$

12. In the given figure, find the value of y



Sol : Ray OP and OQ stands on line AOB

$\therefore \qquad \angle AOQ + \angle QOP + \angle POB = 180^0$[Linear pair axiom]

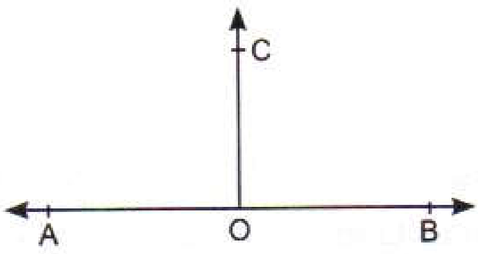
$\Rightarrow \qquad 3y + 40^0 + 2y = 180^0$

$\Rightarrow \qquad 5y = 180^0 - 40^0 = 140^0$

$\Rightarrow \qquad y = \frac{140^0}{5} = 28^0 \Rightarrow y = 28^0$

13. If ray OC stands on line AB such that $\angle AOC = \angle BOC$, then show that $\angle BOC = 90^0$

Sol : Ray OC stands on line AOB



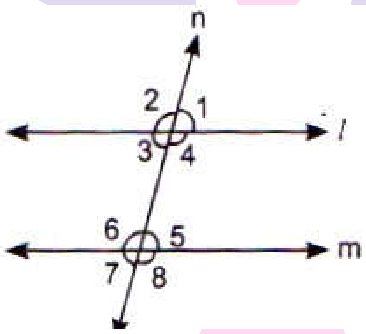
$\therefore$ $\angle AOC + \angle BOC = 180$

$\Rightarrow$ $2\angle BOC = 180^0$ [$\because$ $\angle BOC = \angle AOC$]

$\Rightarrow$ $\angle BOC = 90^0$ Hence Proved.

II. Multiple choice questions 6.2

1. In the given figure $\angle 4$ and $\angle 5$ are known as



(a) alternate interior angles

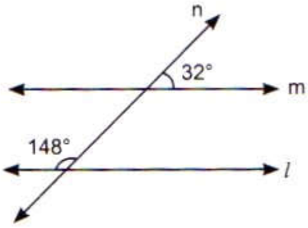
(b) alternate exterior angles

(c) corresponding angles

(d) interior angles on the same side of transversal

Sol : d

2. In the given figure, the relation between line l and line m is



(a) l|| m

(b) l is not parallel to m

(c) lines l and m, intersect when produced

(d) none of these

Sol : a

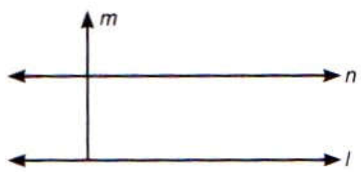
3. Line l is perpendicular to line m and line m is perpendicular to line n, the line l is ____ to line n

(a) parallel

(b) perpendicular

(d) intersecting

(d) none of these

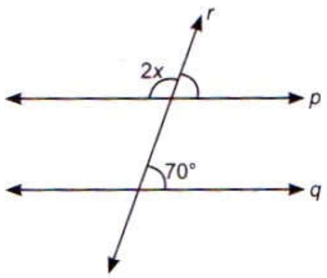


Sol : Given, l ⊥ m and m ⊥ n

So line l is parallel to line n,

∴ correct option is (a)

4. In the given figure, $p||q$, Find the value o x. **[CBSE2010]**



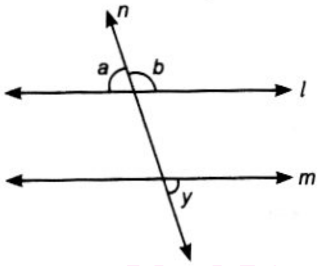
Sol : Since $p \parallel q$ and r is transversal,

∴ angle which the line r makes with line p is 70^0(Corresponding angle)

and $2x + 70^0 = 180^0$ [Linear pair axiom]

$\Rightarrow$ $2x + 110^0 \Rightarrow x + 55^0$

5. **In the given figure, if $l||m$ and $\angle a : \angle b$ = 2 : 3 then find the value of $\angle y$.**



Sol : Given $\angle a : \angle b$ = 2 : 3

$\Rightarrow \quad \frac{a}{b} = \frac{2}{3} = \frac{2k}{3k}$

$\Rightarrow \quad a = 2k \text{ and } b = 3k$

Now, $\angle a : \angle b$ = 180^0 [Linear pair axiom]

$\Rightarrow \quad 2k + 3k = 180^0$

$\Rightarrow \quad 5k = 180^0 \Rightarrow k = 36^0$

$\therefore \quad \angle a = 2k = 2 \times 36^0 = 72^0$

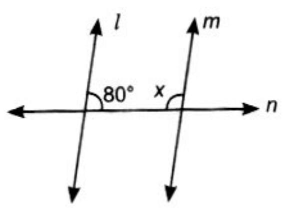
Now, $\angle a$ and $\angle y$ are the alternate exterior angles,

$\therefore \quad \angle a = \angle y \; or \; \angle y = \angle a = 72^0$

6. **If a transversal intersects a pair of lines in such a way that a pair of alternate angles are equal, then what conclusion would you like to draw?**

Sol : By alternate interior angles theorem, we conclude that a pair of lines are parallel to each other.

7. **If a line $l \,||\, m, n$ is a transversal in the given figure, Find the value of x.**

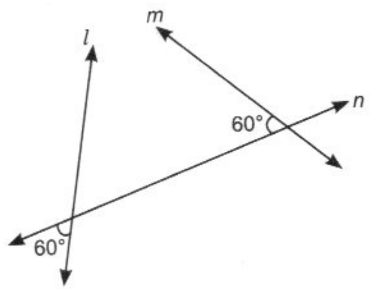


Sol : $l \,||\, m$, and n is transversal. Then sum of pair of interior adjacent angles on the same side of transversal is supplementary.

$\therefore \quad x + 80^0 = 180^0$

$\Rightarrow \quad x = 100^0$

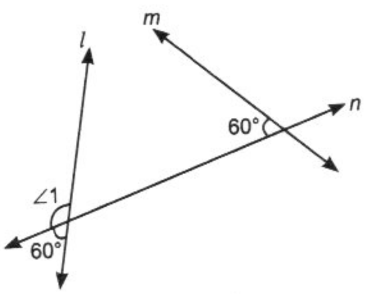
8. Check whether l is parallel to m or not?



Sol : $\angle 1 + 60^0 = 180^0$ [Linear pair axiom]

$\angle 1 = 180^0 - 60^0$

$= 120^0$



So from the figure, the corresponding angles which makes transversal n with l and m are not equal. Hence, l is not parallel to m.

III. Multiple choice questions 6.3

1. What is common between the three angles of a triangle and a linear pair axiom?

(a) angles are equal

(b) in both cases, sum of angles is 180^0

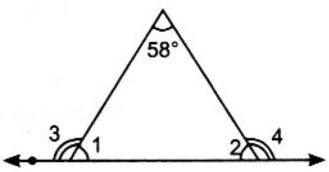
(c) In triangle, there are three angles and in linear pair, there are two angles

(d) None of these

Sol : b

2. In the given figure, $\angle 1 = \angle 2$ then the measurements of $\angle 3$ and $\angle 4$ respectively are

(a) $58^0, 61^0$ (b) $61^0, 61^0$ (c)$119^0, 61^0$+ (d) $119^0, 119^0$



Sol : From the figure, $\angle 1 + \angle 2 + 58 = 180^0$ (Angle sum property of triangle]

But, given $\angle 1 = \angle 2$

So $\angle 1 + \angle 1 + 58^0 = 180^0$

$\Rightarrow \quad 2\angle 1 = 122^0$

$\Rightarrow \quad \angle 1 = \frac{122^0}{2} = 61^0$

So, $\quad \angle 2 = 61^0$

Now, $\quad \angle 3 = 58^0 + \angle 2$

($\because$ Exterior Angle Property)

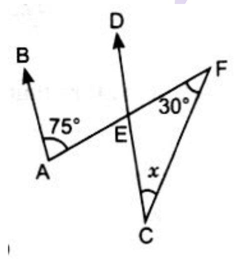
$= 58^0 + 61^0 = 119^0$

Also, $\quad \angle 4 = 58° + \angle 1$ ($\because$ Exterior Angle Property)

$= 58° + 61° = 119°$

$\therefore$ *Correct option is* (*d*)

3. In the given figure, $AB || CD$, the value of x is



a) 45^0 b) 60^0 c) 90^0 d) 105^0

Sol : Given $AB \,||\, CD$,

$\Rightarrow \quad \angle BAE + \angle AED = 180°$

(since Interior angles on the same side of the transversal are supplementary)

$\Rightarrow \quad 75^0 + \angle AED = 180^0$

$\Rightarrow \quad \angle AED = 105^0$

Also $\quad \angle AED = \angle CEF$

($\because$ Vertically opposite Angles)

$\Longrightarrow \qquad \angle CEF = 105^0$

Now in $\triangle CEF$

$\Longrightarrow \qquad \angle CEF + \angle EFC + \angle FCE = 180^0$

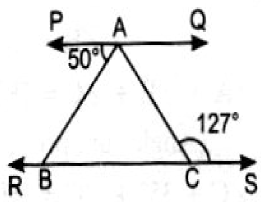
($\because$ Angle sum property of a triangle)

$\Longrightarrow \qquad 105^0 + 30^0 + x = 180^0$

$\Longrightarrow \qquad 135^0 + x = 180^0 \quad \Longrightarrow x = 45^0$

$\therefore \quad Correct\ option\ is\ (a)$

4. In the given figure, $PQ || RS$ and $\angle ACS = 127^0$, $\angle BAC\ is$,



a) 53° b) 77°

c) 50° d) 107°

Sol:

Since PQ || RS, so

$\Longrightarrow \qquad \angle PAC = \angle ACS$

($\because$ Alternate interior angles)

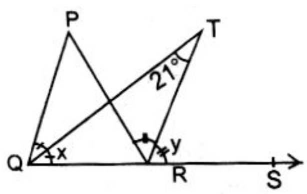
$\Longrightarrow \qquad \angle PAB = \angle BAC = 127^0$

$\Longrightarrow \qquad 50^0 = \angle BAC = 127^0$

$\Longrightarrow \qquad \angle BAC = 77^0$

$\therefore \quad Correct\ option\ is\ (b)$

5. In the given figure, measure of $\angle QPR\ \ is$



a) 10.5^0 b) 42^0 c) 111^0 d) 50^0

Sol : Using exterior angle property, we have

$\angle TRS = \angle QTR + \angle TQR(\ In\ \Delta QTR)$

And $\angle PRS = \angle QPR + \angle PQR(\ In\ \Delta QPR)$

$\Rightarrow \quad 2\angle TRS = \angle QTR + 2\angle TQR$

$\Rightarrow \quad 2(\angle TRS - \angle TQR) = \angle QTR$ .....(i)

Similarly, $2(\angle TRS - \angle TQR) = 2\angle QTR$ .....(ii)

Hence, using (i) and (ii), we get

$\angle QPR = 2\angle QTR$

$= 2 \times 21^0 = 42^0$

$\therefore$ *Correct option is (b)*

6. Number of triangles which can be drawn with angles 42^0, 65^0 *and* $74^0$are

a) one triangle b) two triangles c) many triangles d) no triangle

Sol: d) angle sum cannot be more than 180^0

7. A triangle can have two obtuse angles.

a) True b) False

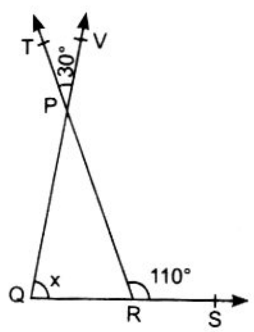
Sol: (b) angle sum cannot be more that 180^0

8. An exterior angle of a triangle is $105^0$and its two interior opposite angles are equal. Each of these equal angles is [NCERT Exemplar]

a) $37\frac{1}{2}^0$ b) a) $52\frac{1}{2}^0$ c) $72\frac{1}{2}^0$ d) 75^0

Sol : b

9. In the given figure, find the $\angle x$



Sol : We have $\angle QPR = \angle TPV$

....(Vertically opposite angles)

$\Rightarrow \quad \angle QPR = 30^0$

From exterior angle theorem,

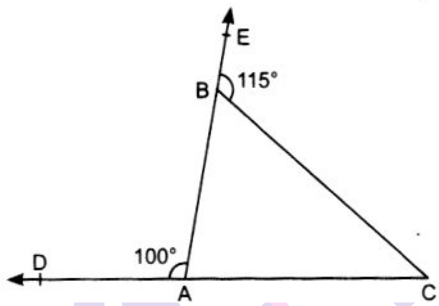
$\angle QPR + \angle PQR = \angle PRS$

$\Rightarrow$ $30^0 + x = 110^0$

$\Rightarrow$ $x = 110^0 - 30^0$

$= 80^0$

10. In the given figure, $\angle EBC = 115^0$ and $\angle DAB = 100^0$. *Find* $\angle ACB$,



Sol : From the figure, $\angle EBC + \angle ABC = 180^0$

[Linear pair axiom]

$\Rightarrow$ $115^0 + \angle ABC = 180^0$

$\Rightarrow$ $\angle ABC = 180^0 - 115^0 = 65^0$

Now, in $\angle ABC$, $\angle BAD = \angle ABC + \angle ACB$

[Exterior angle theorem]

$\Rightarrow$ $100^0 = 65^0 + \angle ACB$

$\Rightarrow$ $\angle ACB = 100^0 - 65^0 = 35^0$

Hence, $\angle ACB = 35^0$

11. The angle of a triangle ABC are in the ratio 2 : 3 : 4 Find the largest angle of the triangle. [CBSE 2016]

Sol : Given $\angle A : \angle B : \angle C = 2 : 3 : 4$

Let $\angle A = 2x, \angle B = 3x, \angle C = 4x$

Using angle sum property of a triangle, we have

$\angle A + \angle B + \angle C = 180^0$

$\Rightarrow$ $2x + 3x + 4x = 180^0$

$\Rightarrow$ $9x = 180^0$

$\Rightarrow \qquad x = 20^0$

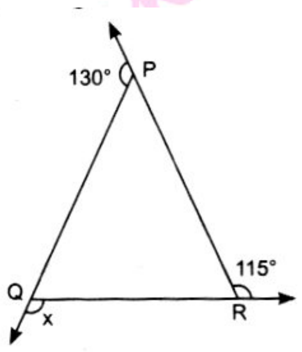
$\therefore \qquad \angle A = 2x = 2 \times 20^0 = 40^0$

$\angle B = 3x = 3 \times 20^0 = 60^0$

$\angle C = 4x = 4 \times 20^0 = 80^0$

Hence largest angle of the triangle is 80^0

12. In the given figure, find the value of x.



Sol : We know that sum of exterior angle of ΔPQR is 360^0

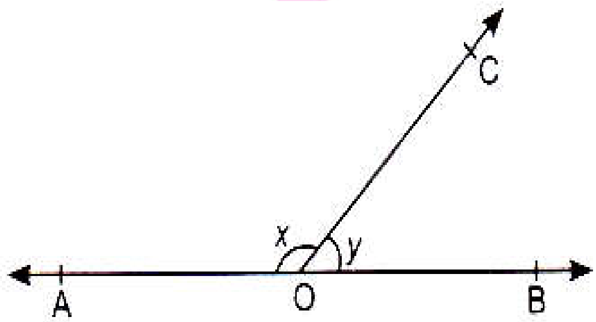
$\Rightarrow \qquad 130^0 + x + 115^0 = 360^0$

$\Rightarrow \qquad 245^0 + x = 360^0$

$\Rightarrow \qquad x = 360^0 - 245^0 = 115^0$

I. Short Answer Type

1. In the given figure, if x is greater than y by one third of a right angle, find the values of x and y.



Sol : $\qquad x = \frac{1}{3} x\, 90^0 + y \qquad$ (Given)

$\Rightarrow \qquad x = 30^0 + y$

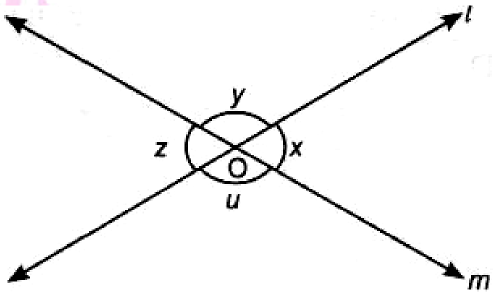
Now, $\angle AOC + \angle BOC = 180^0$ [Linear pair axiom]

$\Rightarrow \qquad x + y = 180^0 \Rightarrow 30^0 + y + y = 180^0$

$\Rightarrow \qquad 2y = 150^0 \Rightarrow y = \dfrac{150^0}{2} = 75^0$

So, $\qquad x = 30^0 + y = 30^0 + 75^0 = 105^0$

2. Lines l and m intersect at O, if x = 45^0, *find y, z and u*.



Sol : $\angle x$ *and* $\angle z$ are vertically opposite angles

$\therefore \qquad \angle x = \angle z = 45^0 \Rightarrow \angle z = 45^0$

But x and y are linear pair angles

So, $\angle x + \angle y = 180^0$ [Linear pair axiom]

$\Rightarrow \qquad 45^0 + \angle y = 180^0 \Rightarrow \angle y = 180^0 - 45^0 = 135^0$

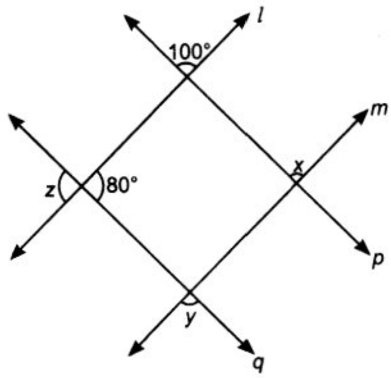
Also, $\angle y$ *and* $\angle u$ are vertically opposite angles

$\therefore \qquad \angle u = \angle y = 135^0 \Rightarrow \angle u = 135^0$

Hence, $\qquad \angle z = 45^0 \;\; \angle y = 135^0$ and $\angle u = 135^0$

II. Short Answer Type

1. Find the value of x and y in the given figure, if $l \,||\, m$ *and* $p \,||\, q$.



Sol : As $l \,||\, m$ *and line p is transversal*

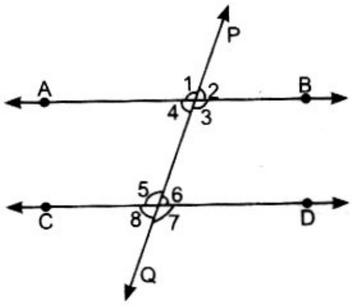
So, $\qquad$ x = 100^0 [Corresponding angles]

Now, $\qquad$ z = 80^0 [Vertically opposite angle

But, $\qquad$ y = 180^0-z [Corresponding angles]

$\therefore \qquad$ y = 180^0- $80^0 = 100^0$

2. In the given figure, $AB \parallel CD, \angle 2 = 120^0 + x$ and $\angle 6 = 6x$. Find the measure of $\angle 2$ and $\angle 6$



Sol : Given $AB \parallel CD$,

$\Rightarrow \quad \angle 2 = \angle 6$ (Corresponding angles)

$\Rightarrow \quad 120^0 + x = 6x \quad [\angle 2 = 120 + x]$

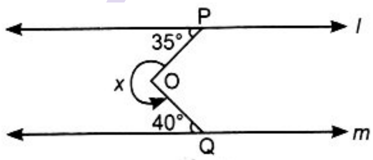
$\Rightarrow \quad 120^0 = 6x - x = 5x$

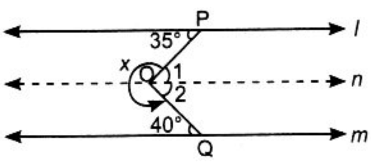
$\Rightarrow \quad x = \frac{120^0}{5} = 24^0$

$\therefore \quad \angle 2 = 120^0 + x = 120^0 + 24^0 = 144^0$

And $\quad \angle 6 = 6x = 6 \times 24^0 = 144^0$

3. In the given figure, if l|| n, find the value of x

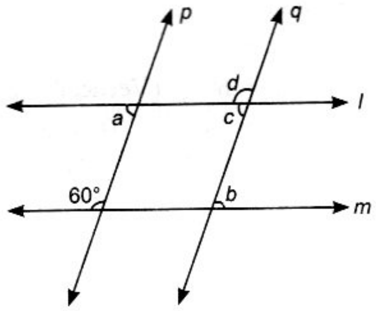




Sol : Draw a line 'n' through O such that $n \parallel l$ and $n \parallel m$.
As $l \parallel n$, OP is transversal.
$\Rightarrow \quad \angle 1 = 35^0$ (Alternate interior angles)
Also, n ∥ m, OQ is transversal
$\angle 2 = 40^0$ (Alternate interior angles)
$\therefore \quad \angle POQ = \angle 1 + \angle 2 = 35^0 + 40^0 = 75^0$
So, $\quad x$ = reflex $\angle POQ$
$= 360^0 - \angle POQ = 360^0 - 75^0 = 285^0$

III. Short Answer Type

1. Lines $l \,||\, m$ and $p \,||\, q$ in the given figure, then find the value of a,b,c, and d.



Sol : Given $l \,||\, m$ and p is transversal

$\Rightarrow \quad a + 60^0 = 180^0$

(Co – interior angles on the same side of transversal)

$\Rightarrow \quad a = 120^0$

But $p||q$ $[Given]$

$\Rightarrow \quad c = a = 120^0$ (Corresponding angles)

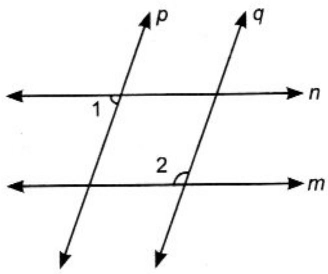
And c = b (Alternate interior angles as $l||m$)

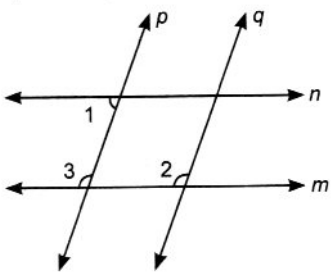
$\Rightarrow \quad b = 120^0$

Also, c + d = 180^0 [Linear pair axiom]

$\Rightarrow \quad d = 180^0 - c = 180^0 - 120^0 = 60^0$

2. In the given figure, $n \,||\, m$ and $p \,||\, q$ of $\angle 1 = 75^0$, prove that $\angle 2 = \angle 1 + \frac{1}{3}$of a right angle.





Sol : Given $\angle 1 = 75^0$

Now, $m||n$ and p is transversal

$\Rightarrow \quad \angle 1 + \angle 3 = 180^0$ (Co – interior angles)

$\Rightarrow \quad 75^0 + \angle 3 = 180^0$

$\Rightarrow \quad \angle 3 = 180^0 - 75^0 = 105^0$

Now $p||q$ and m is transversal

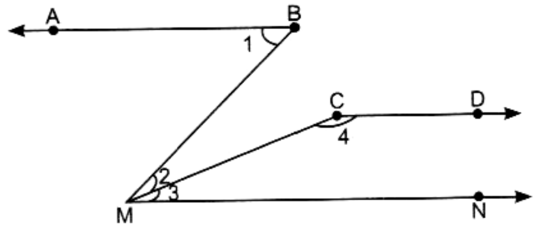
$\Rightarrow \quad \angle 2 = \angle 3 = 105^0$ (Corresponding angles)

$$= 75^0 + 30^0 = 75^0 + \frac{1}{3} \times 90^0$$

$$\angle 2 = \angle 1 + \frac{1}{3} \times right\ angle.$$

Hence, Proved

3. In the given figure, $\angle 1 = 55^0$, $\angle 2 = 20^0$, $\angle 3 = 35^0$ and $\angle 4 = 145^0$. Prove that $AB||CD$



Sol : We have, $\angle BMN = \angle 2 + \angle 3 = 20^0 + 35^0 = 55^0$

$\angle 1 = \angle ABM$

But these are the alternate angles formed by transversal BM on AB and MN.

So, by converse of alternate interior angles theorem,

$AB||MN$ ……(i)

Now $\angle 3 + \angle 4 = 35^0 + 145^0 = 180^0$

This shows that sum of the co-interior angles is 180^0

Hence $CD || MN$ ……(ii)

From (i) and (ii), we have $AB || CD$. Hence proved.

IV. Short Answer Type

1. **Two angles of triangle are equal and the third angle is greater than each of these angles by30^0 Find all the angles of the triangle.**

Sol : Let each of the two equal angles be x. According to the question,

third angle $= x + 30^0$

Now, sum of angles of $\Delta = 180^0$

[Angle sum property of a triangle]

$\Rightarrow \quad x + x + x + 30^0 = 180^0$

$\Rightarrow \quad 3x = 180^0 - 30^0 = 150^0$

$\Rightarrow \quad x = \frac{150^0}{3} = 50^0$

Thus, angles of triangle are 50^0 , $50^0$and 80^0 respectively.

2. **One of the angles of triangle is 75^0, find the remaining two angles if their difference is 35^0.**

Sol : Let in Δ ABC ,$\angle A = 75^0$ and $\angle B - \angle C = 35^0$

$\Rightarrow \quad \angle B = \angle C + 35^0$

Now, $\angle A + \angle B + \angle C = 180^0$

(Angle sum property of a triangle)

$\Rightarrow \quad 75^0 + \angle C + 35^0 + \angle C = 180^0$

$\Rightarrow \quad 110^0 + 2\angle C = 180^0$

$\Rightarrow \quad 2\angle C = 180^0 - 110^0 = 70^0$

$\Rightarrow \quad \angle C = \frac{70^0}{2} = 35^0$

And $\quad \angle B = \angle C + 35^0$

$= 35^0 + 35^0 = 70^0$

3. Prove that if one angle of a triangle is equal to the sum of the other two angles, then the triangle is right angled.

Sol : Given : In ΔABC , $\angle A = \angle B + \angle C$

Now, $\angle A + \angle B + \angle C = 180^0$

(Angle sum property of a triangle)

$\Rightarrow \quad \angle A + (\angle B + \angle C) = 180^0$

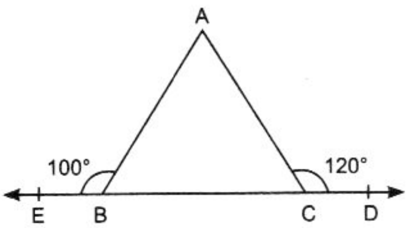
$\Rightarrow \quad \angle A + \angle A = 180^0$

$\Rightarrow \quad 2\angle A = 180^0$

$\Rightarrow \quad \angle A = \frac{180^0}{2} = 90^0$

Hence, with $\angle A = 90^0$ the given triangle is right angled triangle.

4. The exterior angles obtained on producing the base of a triangle both ways are 100^0 *and* 120^0, Find all the angles. **[CBSE 2011]**



Sol : In ΔABC, $\angle ABE + \angle ABC = 180^0$

[Linear pair axiom]

$\Rightarrow \quad 100 + \angle ABC = 180^0$

$\Rightarrow \quad \angle ABC = 180^0 - 100^0 = 80^0$

Similarly, $\angle ACB + \angle ACD = 180^0$

[Linear pair axiom]

$\Rightarrow \quad \angle ACB + 120^0 = 180^0$

$\Rightarrow \quad \angle ACB = 180^0 - 120^0 = 60^0$

Now, again in ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180^0$(Angle sum property of triangle)

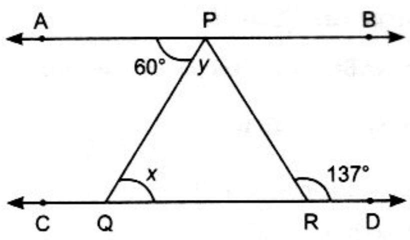
$\Rightarrow \quad 80^0 + 60^0 + \angle BAC = 180^0$

$\Rightarrow \quad \angle BAC = 180^0 - 140^0 = 40^0$

Hence, $\angle BAC = 40^0, \angle ABC = 80^0$

and $\angle ABC = 60^0$

5. **In the given figure, if $AB \parallel CD$ $\angle APQ = 60^0$ and $\angle PRD = 137^0$, then find the value of x and y** [CBSE 2010]



Sol : Given AB || $CD,$

PQ is transversal

$\Rightarrow$ $\angle APQ = \angle PQR$ [Alternate interior angles]

$\Rightarrow$ $60^0 = x$

Again in Δ PQR, exterior angle is ∠PRD

So, ∠ PRD = $\angle PQR + \angle QPR$

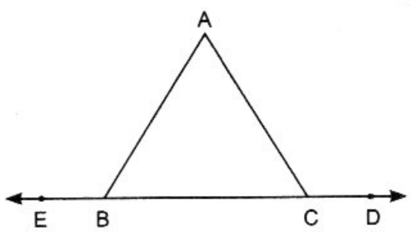
[∵ Exterior angle theorem]

$\Rightarrow$ $137^0 = x + y$

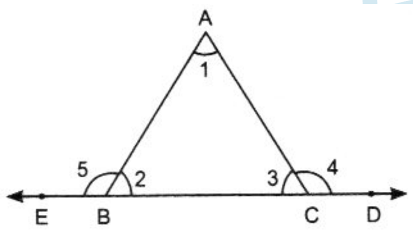
$\Rightarrow$ $137^0 = 60^0 + y$

$\Rightarrow$ $y = 137^0 - 60^0 = 77^0$

6. **In the given figure, side BC of ΔABC is produced in both the directions. Prove that the sum of two exterior angles so formed is greater than 180^0.**



Sol : The exterior angles in the given ΔABC are ∠ABE and $\angle ACD$



To prove ,∠ABE + ∠ACD $> 180^0$

Proof : In ΔABC

$\angle 5 = \angle 1 + \angle 3$ ....(i)

(Exterior angle theorem)

and $\angle 4 = \angle 1 + \angle 2$ .....(ii)

Adding (i) and (ii) we get

$\angle 4 + \angle 5 = \angle 1 + \angle 3 + \angle 1 + \angle 2$

$= \angle 1 + (\angle 1 + \angle 2 + \angle 3)$

$= \angle 1 + 180^0$ [Angle sum property of a triangle]

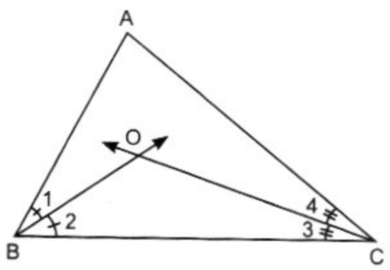
$\Rightarrow$ $\angle 4 + \angle 5 = 180^0$ Hence proved.

V. Short Answer Type

1. In Δ ABC, the bisector of $\angle$B and $\angle$C meets at O. Prove that $\angle$BOC = $90^0 + \frac{\angle A}{2}$

[CBSE 2014]

Sol : Given the bisector of $\angle$B and $\angle$C of ΔABC meets at O as shown in figure.



OB is bisector of $\angle B$

$\Rightarrow$ $\angle 1 = \angle 2 = \frac{1}{2}\angle ABC = \frac{1}{2}\angle B$

Similarly, OC is bisector of $\angle C$

$\Rightarrow$ $\angle 3 = \angle 4 = \frac{1}{2}\angle ACB = \frac{1}{2}\angle C$

Now in ΔABC,

$\angle A + \angle B + \angle C = 180^0$

[Angle sum property of a triangle]

$\Rightarrow \qquad \angle B + \angle C = 180^0 - \angle A$

$\Rightarrow \qquad \frac{1}{2}\angle B + \frac{1}{2}\angle C = 90^0 - \frac{\angle A}{2}$

$\angle 2 + \angle 3 = 90^0 - \frac{\angle A}{2}$ .....(i)

In $\angle$ BOC

$\angle$ OBC + $\angle$ BOC + $\angle$ BCO = 180^0

[Angle sum property of a triangle]

$\Rightarrow \qquad \angle 2 + \angle BOC + \angle 3 = 180^0$

$\Rightarrow \qquad (\angle 2 + \angle 3) + \angle BOC = 180^0$

$\Rightarrow \qquad 90^0 - \frac{\angle A}{2} + \angle BOC = 180^0$

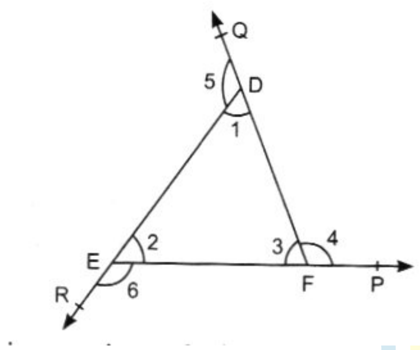
$\Rightarrow \qquad \angle BOC = 180^0 - 90^0 + \frac{\angle A}{2}$

$\Rightarrow \qquad \angle BOC = 90^0 + \frac{\angle A}{2}$

Hence proved.

2. The sides EF, FD and DE of a triangle DEF are produced in order forming three exterior angles DFP, EDQ and FER respectively. Prove that

$\angle$DFP + $\angle$EDQ + $\angle$FER = 360^0



Sol : By using exterior angle theorem, we have

$\angle 4 = \angle 1 + \angle 2$ .... (i)

$\angle 5 = \angle 2 + \angle 3$ .... (ii)

and $\angle 6 = \angle 1 + \angle 3$ .... (iii)

Adding (i), (ii) and (iii), we get

$\angle 4 + \angle 5 + \angle 6 = (\angle 1 + \angle 2) + (\angle 2 + \angle 3) + (\angle 1 + \angle 3)$

$= 2\ (\angle 1 + \angle 2 + \angle 3\ )$

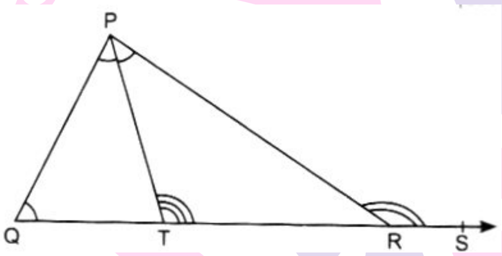
$= 2 \times 180^0 (\because \angle 1 + \angle 2 + \angle 3 = 180^0)$

$= 360^0$

(Angle sum property of a triangle)

$\Rightarrow$ ∠DFP + ∠EDQ + ∠FER = 360^0 Hence Proved.

3. Side QR of Δ PQR is produced to a point S as shown in the figure. The bisector of ∠P meets QR at T. Prove that ∠PQR + ∠PRS = 2 ∠PTR.



Sol : ∠PRS is the exterior of Δ PQR

∴ ∠PRS = ∠QPR + ∠PQR

[Exterior angle theorem]

= 2 ∠TPQ + ∠PQR

Adding ∠PQR on both sides, we get

[PT is bisector of ∠P ∴ ∠TPQ = $\frac{1}{2}\angle QPR$]

∠PQR + ∠PRS = ∠PQR + 2∠TPQ + ∠PQR

= 2 (∠TPQ + ∠PQR) ......(i)

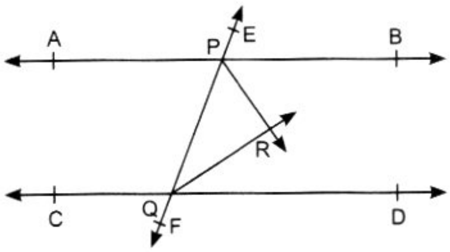
Now in Δ **PTQ,** ∠PTR is exterior angle

∠PTR = ∠TPQ + ∠PQR) ......(ii)

Thus from (i) and (ii), we get

∠PQR + ∠PRS = 2∠PTR Hence proved.

4. **In the given figure, AB and CD are two parallel lines intersected by a transversal EF. Bisector of interior angles BPQ and DQP intersect at R. Prove that ∠PRQ = 90^0**



Sol : Given $AB \parallel CD$ and EF is transversal

$\therefore \quad \angle BPQ + \angle DQP = 180^0$

(Interior angles on the same side of transversal is supplementary)

$\Rightarrow \quad \frac{1}{2}\angle BPQ + \frac{1}{2}\angle DQP = 180^0 \times \frac{1}{2} = 90^0$ .....(i)

Now, PR is the bisector $\angle BPQ$

$\Rightarrow \quad \angle RPQ = \frac{1}{2}\angle BPQ$

and QR is the bisector $\angle DQP$

$\Rightarrow \quad \angle PQR = \frac{1}{2}\angle DQP$

From (i), we have $\angle RPQ + \angle PQR = 90^0$ .....(ii)

InΔPQR, $\angle RPQ + \angle PQR + \angle PRQ = 180^0$

(Angle Sum property of a triangle)

$\Rightarrow \quad 90^0 + \angle PRQ = 180^0$

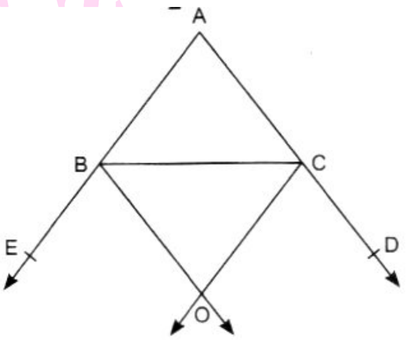
$\Rightarrow \quad \angle PRQ = 180^0 - 90^0 = 90^0$

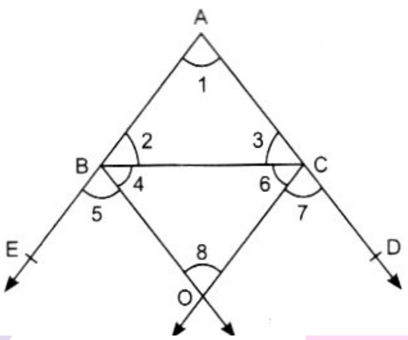
Hence Proved.

1. Long Answer Type

1. In the given figure, bisectors of the exterior angles B and C formed by producing sides AB and AC of ΔABC intersect each other at the point O.

Prove. That $\angle BOC = 90^0 - \frac{1}{2}\angle A$





Sol : Ray BO is the bisector of $\angle CBE$

$\Rightarrow \quad \angle 4 = \angle 5 = \frac{1}{2}\angle CBE$

Now, $\angle 2 + \angle 4 + \angle 5 = 180^0$ [Linear pair axiom]

$\Rightarrow \quad \angle 2 + 2\angle 4 = 180^0 \qquad (\because \angle 4 = \angle 5)$

$\Rightarrow \quad \angle 4 = 90^0 - \frac{\angle 2}{2}$ .....(i)

Similarly, ray OC bisect $\angle BCD$

$\angle 6 = \frac{1}{2}\angle BCD = \frac{1}{2}180^0 - \angle 3$

$= 90^0 - \frac{\angle 3}{2}$ .....(ii)

Now, in $\angle BOC$

$\angle 4 + \angle 6 + \angle 8 = 180^0$

(Angle Sum property of a triangle)

$\Rightarrow\ \left(90^0 - \frac{\angle 2}{2}\right) + \left(90^0 - \frac{\angle 3}{2}\right) + \angle 8 = 180^0$

$\Rightarrow\ \angle 8 = \frac{1}{2}(\angle 2 + \angle 3)$ .....(iii)

Again in Δ ABC

$\angle 1 + \angle 2 + \angle 3 = 180^0$

(Angle Sum property of a triangle)

$\Rightarrow\ \angle 2 + \angle 3 = 180^0 - \angle 1$

Substituting in (iii) we get

$\angle 8 = \frac{1}{2}(180^0 - \angle 1)$

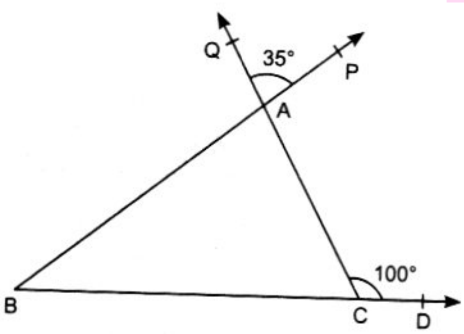
$\Rightarrow \angle 8 = 90^0 - \frac{\angle 1}{2}$

Or $\angle BOC = 90^0 - \frac{\angle BAC}{2}$ $or\ \angle BOC$

$= 90^0 - \frac{1}{2}\angle A$

Hence, proved.

2. Side, BC, CA and BA of triangle Δ ABC produced to D, Q, P respectively as shown in the figure. If $\angle$ACD = 100^0 and $\angle$QAP = 35^0 find all the angles of a triangle. [CBSE 2014]



Sol : We have $\angle$BAC = $\angle$QAP

[Vertically opposite angles]

$\Rightarrow \angle BAC = 35^0$

.... (Given that $\angle$QAP = 35^0)

Also, $\angle$ACB + $\angle$ACD = 180^0

[Linear pair axiom]

$\Rightarrow$ $\angle ACB + 100^0 = 180^0$

$\Rightarrow$ $\angle ACB = 180^0 - 100^0 = 80^0$

In ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180^0$

(Angle Sum property of a triangle)

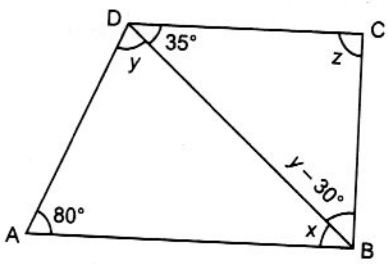
$\angle ABC + 80^0 + 35^0 = 180^0$

$\angle ABC + 115^0 = 180^0$

$\Rightarrow$ $\angle ACB = 180^0 - 115^0 = 65^0$

Hence, $\angle ABC = 65^0$ + $\angle BAC = 35^0$ and $\angle ACB = 80^0$

3. In the given figure, $AB \parallel DC$, $\angle BDC = 35^0$ and $\angle BAD = 80^0$, Find x, y, z



C

Sol : Given $AB \| DC$

BD is transversal

$\Rightarrow$ $x = 35^0$ [Alternate interior angles]

In ΔABD, $\angle ABD + \angle ADB + \angle BAD = 180^0$

(Angle Sum property of a triangle)

$\Rightarrow$ $x + y + 35^0 = 180^0$

$\Rightarrow$ $35^0 + y + 80^0 = 180^0$ $(\because x = 35^0)$

$\Rightarrow$ $y = 180^0 - 115^0 = 65^0$

$\therefore$ $\angle DBC = y - 30^0 = 65^0 - 30^0 = 35^0$

Again in ΔBCD

$\angle DBC + \angle BCD + \angle CDB = 180^0$

(Angle Sum property of a triangle)

$\Rightarrow \quad 35^0 + z + 35^0 = 180^0$

$\Rightarrow \quad z = 180^0 - 70^0 = 110^0$

Hence, $x = 35^0$, $y = 65^0$, $and \ \ z = 110^0$